Math 347: Presentation 1

September 11, 2018

Description: Studying mathematics is a collaborative process and writing proofs should be as well. In math classes after this, you will be expected to explain your solutions to your peers, or present your argument. While that can be done in a formal written proof, as in the homework, it is also important to learn to do this on the board and to be able to talk someone through your ideas to solve a particular problem. The purpose of this assignment is to work on that.

Instructions: Each problem is going to have two groups assigned to them. Monday (9/17) in class, we will pick one of the groups to present the problem on the board. The main point is to have a discussion with the class and understand how the presenting group tried to solve the problem, even if they still don't have a complete solution, we want to understand what they tried and how far they got into solving the problem. The second group that also thought about this problem can help the class to start the conversation, asking pertinent questions.

Groups:

Group 1: Aman Gulrajani, Andreas Ruiz-Gehrt, Bangzheng Li,Daniel Coonley, and Nishant Dalmia.

Group 2: Dongfan Li, Ismail Dayan, Jacob Elling, Ruisong Li, and Tianshu Qu.

Group 3: Adi Budithi, Bryan Ulziisaikhan, Bug Lee, Jingquan Fu, and Zhe Song.

Group 4: David Deng, Houyi Du, Jinjie Wang, Kaiwen Hu, and Vetrie Senthilkumar.

Group 5: Amr Elayyan, Danyu Sun, Jinsoo Oh, Samantha Barrera, and Thomas Varghese.

Group 6: Albert Cao, David Deng, Zhenghong Huang, Zihe Wu and Zhengsan Chang.

Problem division: Groups 1 and 2 should do problem 1), Groups 3 and 4 should do problem 2) and Groups 5 and 6 should do problem 3). While each group will only be asked to present the problem they were assigned, I encourage everyone to read all the problems and give a thought to them, so they are in better position to participate in the discussion.

Problems

1) Consider a function $f : A \to B$ and let $S \subseteq B$, recall that

$$f^{-1}(S) = \{x \in A \mid f(x) \in S\}.$$

Let S and T be subsets of B.

(i) Determine whether $f^{-1}(S \cup T)$ is equal to $f^{-1}(S) \cup f^{-1}(T)$. Justify your claim either with a proof or counter-example.

(ii) Determine whether $f^{-1}(S \cap T)$ is equal to $f^{-1}(S) \cap f^{-1}(T)$. Justify your claim either with a proof or counter-example.

2) Let $f : \mathbb{R} \to \mathbb{R}$ be a function.

(i) Prove that f can be expressed in an unique way as follows:

$$f(x) = g(x) + h(x),$$

where g and h are functions from $\mathbb{R}$ to itself, satisfying:

$$h(x) = -h(-x), \ \ \forall x \in \mathbb{R}, \quad \text{and} \quad g(x) = g(-x) \ \ \forall x \in \mathbb{R}.$$

(ii) If $f(x)$ is a polynomial, express g and h in terms of the coefficients of f.

3) Consider three circles in the plane in general position, i.e. intersecting as shown in the picture below. Each bounded region contains a token that is white on one side and black on the other side. At each step, we an either (a) flip all four tokens inside one circle, or (b) flip the tokens showing white inside one circle to make all four tokens in that circle to show black. From the starting configuration with all tokens showing black, can we reached the configuration indicated in the figure?